Ungrounded Self-Reference: The Solution to the Liar Paradox?

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Introduction

Adequate solution of the Liar Paradox must satisfy three key criteria: 1. it must apply to all forms of the Liar Paradox; 2. it is not a necessary component of non-paradoxical and meaningful sentences; 3. there must be good reason to adopt the solution beside the fact it works for the Liar Paradox, that is, it must not be ad hoc. I will argue these criteria are satisfied by diagnosing ungrounded self-reference as the problem which generates the Liar Paradox. My account is a development of Michael Clark's but adds an important distinction between grounded and ungrounded self-reference which is necessary for a satisfactory solution to the Liar Paradox.

In §1 I will give a brief explanation of the Liar Paradox. In §2 I will argue that a simple ban on self-reference is not a satisfactory solution. In §3 I will describe Clark’s theory and explain that it is unsatisfactory because it fails to satisfy criterion 2 above, in particular, it forces us to accept meaningful and non-paradoxical sentences are meaningless. In §4 I develop my account of the distinction between grounded and ungrounded self-reference and argue that this development satisfies criterion 2. In §5 I argue that the notion of ungrounded self-reference can also satisfy criterion 1 by defusing all forms of the Liar Paradox. In §6 I argue that ungrounded self-reference satisfies criterion 3 as it is based on common linguistic characteristics and is not ad hoc. I then conclude that taking ungrounded self-reference as the characteristic which allows for the Liar Paradox, we have satisfied
criteria 1-3 and found a solution to the Liar Paradox.

The Liar Paradox

L1: L1 is false.

Above we have the sentence L1, which is the sentence, ‘L1 is false’. This is the Liar sentence. It is a sentence which says of itself that it is false. So the question is, L1 true or false? If L1 is true, then its true that L1 is false, so it’s false. If L1 is false, then it's false that L1 is false, so it’s true. Thus L1 is true iff L1 is false. This is paradoxical because we assume the principle of bivalence, that every sentence is either true or false (and couldn’t be both). L1 entails that L1 is both true and false, contradicting the principle of bivalence. I think a clear way to set out this paradox is a bit more formally, similar to a way in which Rescher does¹.

Let ‘L1’ stand for ‘L1 is true’. Let ‘¬L1’ stand for ‘L1 is false’:

1. \(L1 \leftrightarrow \neg L1\) (Inferred from the sentence L1)
2. \(L1 \rightarrow \neg L1\) (From 1)
3. \(\neg L1 \rightarrow L1\) (From 1)
4. \(\neg L1\) (From 2)
5. \(L1\) (From 3)
6. Contradiction (4,5)

We don’t want to accept that L1 is both true and false, and this is entailed if either truth value is assigned to it. So we must find a good reason to consider it incapable of being either. We have to uncover the characteristic in the Liar Paradox which makes it paradoxical, diagnosing the issue which allows the Liar Paradox to

A Ban on Self-Reference

One attempt to diagnose the issue with L1 is to find the problem in self-reference: sentences which self-refer in the way that L1 does are illegitimate. Some have held that any sentence which says something of itself is meaningless, that is, it is not capable of having or lacking a truth value.\(^2\)

Whilst this is enough to stop the Liar Paradox, as noted by Barwise and Etchemendy, a simple ban on self-reference cannot be the solution. For example, a simple ban was suggested by Jorgenson who argued that self-referential sentences are illegitimate because they are syntactically ill-formed. There isn’t space to discuss it in detail here but this solution is unsatisfactory because it fails to defuse all forms of the Liar Paradox.\(^4\)

Furthermore a simple ban on self-reference would result in banning sentences which are not paradoxical. Yablo offers this example: ‘So dear Lord, to Thee we raise this, our hymn of grateful praise’. This sentence is self-referential but it is in no way paradoxical and certainly seems meaningful. Consider the sentence, ‘This sentence contains five words’. This sentence is self-referential but also true. Could

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\(^2\) I understand there are different accounts of what it is for a sentence to be ‘meaningful’ but for the purposes of this paper we will use this definition.


\(^4\) Jorgenson, J., ‘Some Reflections on Reflexivity,’ *Mind*, Vol. 62, No. 247 (Jul., 1953), pp. 289-300: Jorgenson discussed the liar in the form, ‘This sentence is false’. He argued that the predicate ‘is false’ cannot be meaningfully applied to the subject ‘this sentence’. This however fails to defuse a liar of the form of the sentence L1 above, and the pair of sentences A and B below, all of which would be syntactically well formed on Jorgenson’s account.

the problem with the liar really be its self-referential nature when we have clear examples of meaningful self-referential sentences which pose no threat? As we said in the introduction, a satisfactory solution must defuse all forms of the Liar Paradox, without excluding meaningful sentences.

Clark on Self-Reference and Indexicality

Clark’s work gives an account of self-reference which makes an important distinction between sentences and propositions, which arises because of indexicality. A word or phrase is indexical if its meaning varies in different contexts. A proposition is something a sentence expresses -- it could perhaps be considered the thought or content of a sentence. It is important to make this distinction because two different sentences can express the same proposition:

\begin{enumerate}
\item[(1)] (Said by Bertrand): ‘I’m hungry.’
\item[(2)] (Said by me): ‘Bertrand is hungry.’
\end{enumerate}

Both (1) and (2) express the proposition that ‘Bertrand is hungry’. Similarly the same two sentences can express different propositions:

\begin{enumerate}
\item[(3)] (Said by me): ‘I’m hungry.’
\item[(4)] (Said by you): ‘I’m hungry.’
\end{enumerate}

Both (3) and (4) express different propositions, (3) expressing that I am hungry and (4) expressing that you are hungry.

Clark argues that self-referential sentences fail to express any genuine proposition at all. A sentence which fails to express a proposition is literally meaningless. Unlike other approaches, such as Jorgenson’s which argue self-referential sentences are syntactically ill-formed, Clark is suggesting they are...
semantically ill-formed. One of his examples will help illuminate this:

L3: L3 expresses a false proposition.
L4: L3 expresses a false proposition.

L3 and L4 are the same sentence. L3 would be a Liar Paradox, true iff it is false. However on Clark’s account, as it is self-referential, it fails to express any proposition at all and is thus meaningless. L4, not being self-referential, does indeed express a proposition. It is the falsehood that L3 expresses a false proposition (false because L3 fails to express any proposition at all).

Clark suggests the reason self-referential sentences fail to express a proposition could be their lack of grounding. A sentence is grounded iff there is something about the world that makes it true (or false). Sentences like L1 are ungrounded as the only thing about the world that would determine if L1 is true is if L1 is true. This point is essential to the solution; however Clark leaves it underdeveloped and does not offer a clear enough account of what it is to be grounded.

As it stands it requires we dismiss all self-referential sentences as ungrounded, and thus meaningless. Let’s consider sentence F:

F: The sentence F contains six words.
F is surely a true sentence. Yet on Clark’s account it is ungrounded and thus fails to express a proposition. The same would apply to Yablo’s example above and any self-referring sentence. This shows Clark has failed to satisfy criterion 2. He has the benefit of defusing the Liar Paradox (at least in all forms considered so far) but it will cost us unproblematic sentences which seem meaningful. However this problem can be avoided while keeping the benefits of Clark’s account by developing the

notion of ungrounded self-reference.

Ungrounded Self-Reference

A sentence is grounded if there is something about the world which makes it true. The sentence is the *truth bearer* and the fact about the world is that sentence’s *truth maker*. We’ll make this clearer with an example from Sainsbury. Consider sentence S.

S: Snow is white

S is true, and is the truth bearer. S is made true by the fact about the world that snow is indeed white, which is the truth maker. As noted by Sainsbury, it seems the truth of a sentence depends on something outside itself. However we will require a more fine-grained understanding of grounding. A sentence is grounded iff its truth value depends entirely upon something other than its own truth value. Thus a sentence will be illegitimate if its truth value relies at least partially on its truth value. The definition also allows that a sentence can have properties other than its truth value which can be legitimate truth makers, for example, the number of words in the sentence or the language it is written in. On this definition we can see that S is clearly a grounded sentence.

The only thing about the world which could make L1 true is if L1 is false. The truth value of L1 relies upon its own truth value. This is to say that the sentence L1 is not only the truth bearer but its truth value is also its own truth maker. This is illegitimate. Thus L1 is an example of *ungrounded* self-reference. As such, self-referential liar sentences are semantically ill-formed and fail to express a proposition. This renders them meaningless.

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Our definition would also deal with Liar Paradoxes which involve more than one sentence. Take an example of Rescher’s: ‘Every sentence on this page is false’. The truth maker of that sentence is the truth value of every sentence on this page which includes its own truth value. As such its truth value at least partially relies upon its own truth value -- discovering its truth value would first require discovering its truth value. It is an example of ungrounded self-reference and must be considered meaningless.

Furthermore, in order to allow sentences such as F or Yablo’s earlier example, we must make a distinction between ungrounded self-reference and grounded self-reference. Reconsider F. It seems whether F is true relies on a fact about the sentence F, and so in a sense F is its own truth maker. However according to our more fine-grained definition of grounding, the truth of F does not depend on the truth of F. It depends on a different property of F, namely, how many words are in F. So we can say F is an example of grounded self-reference and succeeds in expressing a proposition; F refers legitimately to a property of itself which is not its truth value. We can say the same of Yablo's sentence, ‘So dear Lord to Thee we raise, this our hymn of grateful praise.’ The truth maker of this sentence is not its own truth but whether we do truly raise our hymn. Thus this is another example of grounded self-reference.

What I have suggested is consistent with Clark’s account but strengthens it. By making the distinction between grounded and ungrounded self-reference, we can rescue unproblematic sentences. This satisfies criterion 2 above. I will now turn to

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10 'F: The sentence F has six words.'

11 'So dear Lord, to Thee we raise this, our hymn of grateful praise.'
showing that it satisfies criterion 1.

**Ungrounded Self-Reference as necessary and sufficient for the Liar Paradox**

L5: *L5 does not express a proposition.*

L5 self-refers in an ungrounded way. To discover the truth value of L5 we must first discover its truth value. Thus L5 doesn’t express a proposition. But doesn’t that make what L5 says true? We can feel the familiar Liar Paradox appearing again.

As noted by Clark, this is where the appeal to indexicality becomes important\(^\text{12}\). As we saw with (3) and (4) it was clear that the same sentence can express different propositions. L5 fails to express any proposition at all. We can legitimately re-use the sentence L5 to state that L5 doesn’t express a proposition. To make the point clearer, let's re-label the sentence as Clark does:

L6: *L5 does not express a proposition.*

It is clear we can now use L6 to express the proposition that L5 does not express a proposition even though L5 and L6 are the same sentence.

Sainsbury argues that this treatment would fail against a new sentence\(^\text{13}\): P: *No use of this very sentence expresses a true proposition.*

The idea is that we couldn’t have a sentence which uses the words of P to express the true proposition that P doesn’t express a proposition. (This is how we dealt with L5.) Clark argues that P fails to pose a problem\(^\text{14}\) because P doesn’t have to be interpreted as referring to itself. One might correctly utter P while referring to L1 according to Clark. It is possible to use P to refer to other sentences but if uttered


self-referentially it is false -- there are ways to use P to express a true proposition. Any problem posed by P can be avoided more simply. Another sentence can meaningfully assert that P doesn’t express a proposition and as that wouldn’t require using the same string of words as P, P fails to pose a problem. For example this could be done by:

P1: P fails to express a true proposition.

There are many variants of strengthened liar cases; another example might be ‘This sentence is not truth apt’\(^\text{15}\). As with L5 and L6, these sentences can always be dealt with by indexicality A liar sentence can be meaningfully described by another sentence which states it is meaningless in which case the describing sentence truly expresses a proposition and the liar sentence expresses no proposition at all on account of ungrounded self-reference.

It seems ungrounded self-reference can deal with all self-referential liar paradoxes. However there seem to be forms of the Liar Paradox which do not involve self-reference; take this example from Sainsbury\(^\text{16}\):

A: (said by α on Monday) ‘Everything β will say Tuesday is true.’

B: (said by β on Tuesday) ‘Everything α said on Monday was false.’

Pairs of sentences in the style of A and B are often cited as an example of a non-self-referential Liar Paradox. But is it correct to say these sentences aren’t self-referential? How might we determine the truth value of A? It would be by determining the truth value of B. How might we determine the truth value of B? It would be by determining the truth value of A. So to determine the truth value of A we must first determine the truth value of A. The same is true of B. Both are

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indirectly self-referential, and ungrounded: Their truth values can be only determined by their own truth values. Thus, neither sentence A nor B expresses a proposition.

Another example of an attempted formulation of a non-self-referential Liar Paradox is Yablo’s Paradox\(^{17}\). Yablo suggested that self-reference wasn’t necessary for the Liar Paradox in an infinite chain of sentences where

\[(S1) \text{For all } k > 1, Sk \text{ is untrue;}
\]
\[(S2) \text{for all } k > 2, Sk \text{ is untrue;}
\]
\[(S3) \text{for all } k > 3, Sk \text{ is untrue}...
\]

Each sentence claims all the sentences which follow it are false. If S1 is true then S2, and all the following, are false. But if S2 is false then all the following sentences would be true, making S1 false. Thus, S1 is true iff it is false. Yablo claimed this Liar Paradox is not self-referential. Sadly there is not space to discuss in full detail here but I will offer a short explanation of my solution\(^{18}\).

Clark argues that each of the sentences is actually implicitly self-referential\(^{19}\). S1 claims for all K greater than 1, that Sk is untrue. This requires that S1 refer to itself, namely, by means of ‘for all K >1’. So S1 could be rephrased as, ‘For all sentences following this sentence….’ And if they are self-referential sentences then they fail to express propositions.

Clark had no reason to go further as he did not have the distinction between grounded and ungrounded self-reference but this distinction is necessary to avoid


other problems. I think all sentences in Yablo’s paradox are examples of ungrounded self-reference. S1 is true if all the following sentences are false. Whether we classify S2 as true or false depends on the truth value of every following sentence. But we might have reason to think the truth value of S2 depends partially on the truth value of S1 because if S1 is true then S2 is false, that is, a necessary condition of the truth of S2 is the falsity of S1. So the truth of S1 relies partially on its own truth value. If this is correct, it applies to every sentence in the chain. Consequently, they are ungrounded and implicitly self-referential and thus fail to express propositions.

My account so far has been able to defuse all suggested forms of the Liar Paradox. This shows that all Liar Paradoxes are examples of ungrounded self-reference. Our solution has satisfied criterion 1. Also only sentences with ungrounded self-reference get discarded as paradoxical and meaningless while such as F\(^{20}\) are allowed as meaningful. This shows that ungrounded self-reference is also sufficient for rendering a sentence meaningless. Our solution has satisfied criterion 2. I now argue that this solution can satisfy criterion 3, it is not an ad hoc solution.

Is ungrounded self-reference an ad hoc solution?

A final worry for this kind of position might be that it is simply too weird. It may seem counter-intuitive that the same string of words can be paradoxical in one context, like L5, and then express a true proposition in a different context, like L6\(^{21}\). Has all of this been engineered just to solve the Liar Paradox?

I submit that this is not the case. Sentences such as (3) and (4) above, that is, 'I am hungry' said by you versus said by me, are everyday examples of indexicality.

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\(^{20}\) 'F: The sentence F contains six words.'

\(^{21}\) 'L5: L5 does not express a proposition.' 'L6: L5 does not express a proposition.'
They are the same sentence but its truth depends entirely upon context: on who says it and whether they are truly hungry. Many more examples can be found of the effect of context upon sentences. If Descartes stood before me and said, ‘I don’t exist,’ this would be necessarily false as Descartes must exist to say anything. However, if I said, ‘Descartes doesn’t exist,’ the truth of the sentence would depend on whether Descartes does exist or not. It would appear that both Descartes and I were expressing the same proposition with our sentences however his is necessarily false whereas mine is a contingently true or false. Or take the problem of humility. Imagine that Descartes were humble. I could correctly say he is humble but it would impossible for Descartes to say it since his saying would mean that he wasn’t humble. Thus it should be clear that context is important for the truth values of a sentence. We can see that the context renders L5 an example of ungrounded self-reference and thus meaningless while allowing L6 to be true.

Conclusion
I have tried to demonstrate that ungrounded self-reference is necessary and sufficient for the Liar Paradox. By distinguishing grounded and ungrounded self-reference we have been able to defuse all Liar Paradoxes while retaining potentially meaningful self-referential sentences. Furthermore I have argued that this is by no means an ad hoc solution. It is certainly reasonable to suppose the meanings of sentences vary in different contexts. I thus conclude that the solution of the Liar Paradox lies in disregarding all ungrounded self-referential sentences as failing to express genuine propositions.

Bibliography
Rescher, N., *Paradoxes: Their Roots, Range and Resolution*, Open Court (2001)